

Ważne wzory związane z całkami

$$1. \int f \cdot g' dx = f \cdot g - \int f' \cdot g dx$$

$$2. \int f(x) dx = \int f[\psi(t)] \psi'(t) dt$$

$$3. \int f(ax+b) dx = \frac{1}{a} F(ax+b), \quad F' = f$$

$$4. \int \frac{f'}{af+b} dx = \frac{1}{a} \ln|af+b|$$

$$5. \int \frac{f'}{\sqrt{af+b}} dx = \frac{2}{a} \sqrt{af+b}$$

$$6. \int \frac{f'}{(af+b)^2} dx = -\frac{1}{a(af+b)}$$

$$7. \int \frac{f'}{a^2 f^2 + b^2} dx = \frac{1}{ab} \operatorname{arctg} \frac{af}{b} ; 7'. \int \frac{f'}{a^2 f^2 - b^2} dx = \frac{1}{2ab} \ln \left| \frac{af-b}{af+b} \right|$$

$$8. \int \frac{f'}{af^2+bf} dx = \frac{1}{b} \ln \left| \frac{f}{af+b} \right|$$

$$9. \int \frac{f' dx}{\sqrt{a^2 - f^2}} = \operatorname{arcsin} \frac{f}{a}$$

$$10. \int \frac{f' dx}{\sqrt{f^2 + a}} = \ln |f + \sqrt{f^2 + a}|$$

$$11. \int \frac{f dx}{(f+a)(f+b)} = \frac{a}{a-b} \int \frac{dx}{f+a} - \frac{a}{a-b} \int \frac{dx}{f+b} ; a \neq b$$

$$12. \int \frac{f dx}{(f+a)^2} = \int \frac{dx}{f+a} - a \int \frac{dx}{(f+a)^2}$$

$$13. \int \frac{f^2 dx}{f^2 - a^2} = \frac{1}{2} \int \frac{f dx}{f-a} + \frac{1}{2} \int \frac{f dx}{f+a}$$

$$14. \int R(\sin x, \cos x) dx = \int R\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right) \frac{2du}{1+u^2} \quad \text{podst. } u = \operatorname{tg} \frac{x}{2}$$

$$15. \int R(\sin^2 x, \cos^2 x, \sin x \cos x) dx = \int R\left(\frac{u^2}{1+u^2}, \frac{1}{1+u^2}, \frac{u}{1+u^2}\right) \frac{du}{1+u^2} \quad \text{podst. } u = \operatorname{tg} x$$